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★**Universalgeschichte der Zahlen.** (German) **[Universal history of numbers]**

Translated from the French by Alexander von Platen.

Campus Verlag GmbH, *Frankfurt am Main*, 1986. 580 *pp*. DM 78.00. *ISBN* 3-593-33666-9

Two initial parts of the book (c. one quarter of the whole) deal with general aspects of "numerical culture": the conceptual foundations of human counting ability, the construction of different number systems (decimal, vigesimal, sexagesimal), and concrete representations of numbers and tools for reckoning (finger reckoning, carving stocks, calculi, quipu and related systems, coins, and abaci). The focal interest of the book, however, is the development of written notations for positive integers from the beginnings until the establishment of the "Arabic" numerals in Western Europe. This subject is covered in depth and impressive breadth, mainly on the basis of an extensive knowledge of the secondary literature but also on personal information from and cooperation with prominent researchers in many fields; anyone interested but not already an expert in (e.g.) early Indochinese traces of the Indian place value system, the number notation used in the Kingdom of Juda, or the variants of the Arabic letter notation for numbers will find much information and many valuable bibliographical references. A listing of all the "numerical cultures" dealt with would be meaningless. Let it only be said that all those normally taken into account (Egypt, Mesopotamia, China, India, Greece, Rome, Aztecs, Maya, and Hebrew and Arabic systems) are discussed; furthermore, the many different systems used, e.g., in Mesopotamia and in China are discussed, not only the one or two systems found in most broad expositions.

The treatment of the material is organized according to systematic rather than strictly chronological criteria, notations following similar principles being used to elucidate each other and as material for the construction of general categories. At the same time, however, the main number notations are presented in the context of general writing systems, and often also in a broader cultural context (a final 10-page chronological table lists the developments of writing and numerical notations in parallel columns). The "history" of the title is thus fully justified.

The exposition is fluent and entertaining, and this richly illustrated book can thus be used both as a popularization of a field of interest to many mathematicians and mathematics teachers, and as a professional tool. It possesses many of the advantages which may be found in an encyclopedic work. At the same time, it is marked by those weaknesses which inevitably are there. Though most sources used are up-to-date and reliable and used with sound critical sense, some errors are taken over uncritically from outdated, superficial or mistaken literature, and some sources are misinterpreted. On p. 502, for instance, a recreational problem is ascribed to the *Līlāvatī*, which is claimed to be a theological treatise from the 8th century; in fact, the problem is not found in Bhāskara II's 12th-century *Līlāvatī* but in a later commentary to that work, and also in Srīdhara's 9th-century arithmetical treatise *Pāṭīgaṇita sāra*. On p. 480 Tobias Dantzig is quoted with approval for the statement that in the 15th century multiplication meant nothing but successive doubling and division nothing but halving. On p. 423 a cuneiform text from Susa is "quoted" in a way which

has nothing to do with either the cuneiform autograph, the transcription or the translation (if the real text had been quoted, nothing would have been left of the argument in question).

Few of the errors affect the value of the book as a popularization; the reader who uses it as support for professional work should, on the other hand, take care and check critical points.

It should be observed that fractions and methods of calculation are not dealt with systematically but only mentioned in a few places where they become relevant for the line of argument.

{The French original has appeared [Editions Seghers, Paris, 1981]. See also the following review.}

Reviewed by *Jens Høyrup*